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13MCA12

First Semester MCA Degree Examination, Dec.2015/Jan.2016
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Write the following in symbolic form and establish if the argument is valid: If A gets the supervisor's position and works hard, then he will get a raise. If he gets a raise, then he will buy a new car. He has not bought a new car. Therefore A did not get the supervisor's position or he did not work hard. **(05 Marks)**

b. Verify the following without using truth tables:
$[(p \to q) \wedge (\neg r \vee s) \wedge (p \vee r)] \quad \therefore \neg q \to s$ **(05 Marks)**

c. Define Tautology. Show that $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology by constructing a truth table. **(05 Marks)**

d. Show that the following argument is invalid by giving a counter example:
$[(p \wedge \neg q) \wedge (p \to (q \to r))] \to \neg r$ **(05 Marks)**

2 a. Verify if the following is valid:
$\forall x[p(x) \vee q(x)]; \quad \exists x \neg p(x)$
$\forall x[\neg g(x) \vee r(x)]$
$\forall x[s(x) \to \neg r(x)] \quad \therefore \exists x \neg s(x)$ **(05 Marks)**

b. Prove that for all real numbers x and y, if $x + y > 100$, then $x > 50$ or $y > 50$. **(05 Marks)**

c. Determine if the argument is valid or not. All people concerned about the environment, recycle their plastic containers. B is not concerned about the environment. Therefore, B does not recycle his plastic containers. **(05 Marks)**

d. Negate and simplify: i) $\forall x[p(x) \wedge \neg q(x)]$, ii) $\exists x[(p(x) \vee q(x)) \to r(x)]$. **(05 Marks)**

3 a. If N is a set of positive integers and R is the set of real numbers, examine which of the following set is empty:
i) $\{x / x \in N, \quad 2x + 7 = 3\}$
ii) $\{x / x \in R, \quad x^2 + 4 = 6\}$
iii) $\{x / x \in R, \quad x^2 + 3x + 3 = 0\}$ **(04 Marks)**

b. Let $S = \{21, 22, 23, \ldots 39, 40\}$. Determine the number of subsets A of S such that:
i) $|A| = 5$
ii) $|A| = 5$ and the largest element in A is 30.
iii) $|A| = 5$ and the largest element is atleast 30.
iv) $|A| = 5$ and the largest element is at most 30.
v) $|A| = 5$ and A consists only of odd integers. **(10 Marks)**

c. Define power set with example. Prove that if a finite set A has n elements then power set of A has 2^n elements. **(06 Marks)**

4 a. Prove by mathematical induction that every positive integer $n \geq 24$ can be written as a sum of 5's and/or 7's. **(08 Marks)**

b. Find an explicit definition of the sequence defined recursively by $a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \geq 2$. **(06 Marks)**

c. Solve the first order recurrence relation $a_1 = 7a_{n-1}$, $n \geq 1$ given that $a_2 = 98$. **(06 Marks)**

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5 a. For any non empty sets A, B, C, prove the following:
i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
ii) $A \times (B - C) = (A \times B) - (A \times C)$ **(08 Marks)**

b. Define one-one and onto function. Let $f : Z \rightarrow z$ (set of integers) be defined by $f(a) = a + 1$, $\forall a \in z$ find whether f is one to one or onto or both or neither. **(06 Marks)**

c. ABC is an equilateral triangle whose sides are of length 1 cm each. If we select 5 points inside the triangle, prove that atleast two of these points are such that the distance between them is less than ½ cm. **(06 Marks)**

6 a. Let A = {1, 2, 3, 4} and let R be the relation on A defined by xRy if and only if x divides y. Find digraph of R and list in-degree and out-degree of all vertices. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 5}. Define a relation R on A × A by $(x_1, y_1) R (x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$. Verify that R is an equivalence relation on A × A. **(06 Marks)**

c. Let A = {1, 2, 3, 4, 6, 12}. On A, define the relation R by aRb if and only if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation. **(08 Marks)**

7 a. Explain Konigsberg bridge problem. **(06 Marks)**

b. Define isomorphism and show that the following graphs are isomorphic.

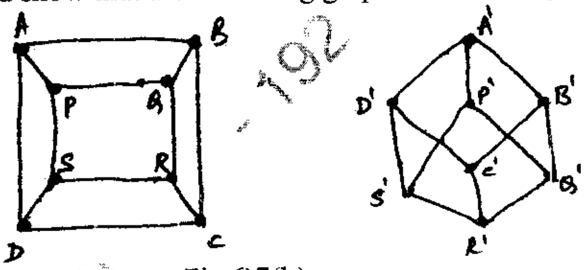

Fig.Q7(b) **(06 Marks)**

c. Define Hamilton cycle. How many edge-disjoint Hamilton cycles exist in the complete graph with seven vertices? Also draw the graph to show these Hamilton cycles. **(08 Marks)**

8 a. Show that the complete bipartite graph $K_{3,3}$ is non-planar. **(06 Marks)**

b. Explain the steps in the merge sort algorithm. **(06 Marks)**

c. Define spanning tree of weighted graph and using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown below:

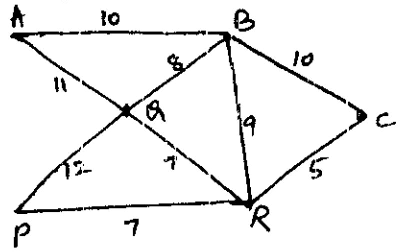

Fig.Q8(c) **(08 Marks)**

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